A COMPARISON BETWEEN TWO MULTIATTRIBUTE DECISION METHODOLOGIES USED IN CAPITAL INVESTMENT DECISION ANALYSIS

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ABSTRACT

Two multicriteria decision methodologies applied to evaluating capital investments are the Analytic Hierarchy Process (AHP) and the Non-Traditional Capital Investment Criteria (NCIC) model. In this paper we demonstrate that a mathematical relationship exists between these two models. In particular, a data set obtained by one method can be mapped into an equivalent data set obtained using the other method. It is suggested that this offers an opportunity of empirically assessing decision makers' judgmental capabilities under varying data collection methods. An example problem illustrates the manner in which such comparisons can be made.

INTRODUCTION

Many studies have been done on multiattribute decision models that are used to evaluate manufacturing technologies. Saaty [19] introduced The Analytic Hierarchy Process (AHP), which is a hierarchical approach to modeling decision problems in general. It has been subsequently adapted to capital budgeting decision making and has become a standard approach to evaluating capital investment alternatives with difficult to quantify criteria (Canada and Sullivan [10]; Canada, Sullivan and White [11]).

Boucher and MacStravic [9] developed Non-Traditional Capital Investment Criteria (NCIC), a methodology that also combines easy-to-quantify and difficult-to-quantify benefits and costs of technology into evaluating capital investments in manufacturing. The methodology takes advantage of some of the techniques of AHP. It was conceived as an alternative to AHP because of certain difficulties found in the AHP methodology when applied to capital investments.

Although the techniques used in the two methods are quite similar, there are differences in the manner in which the decision problem is represented to the decision maker. Both methods have as an objective the "Best Overall Alternative" at the top level of the hierarchy. The Analytic Hierarchy Process then divides the hierarchy into two separate hierarchies, one for benefits and the other for costs (Figure 1). The reason for this is that AHP differentiates between benefits and costs and handles them differently in the pairwise comparison process. Each hierarchy is then divided into major categories that define the economic performance of the system (level 2). These categories are further divided into the unquantifiable as well as the quantifiable criteria (level 3). At the final level of the hierarchy we have the mutually exclusive alternatives that are under consideration.

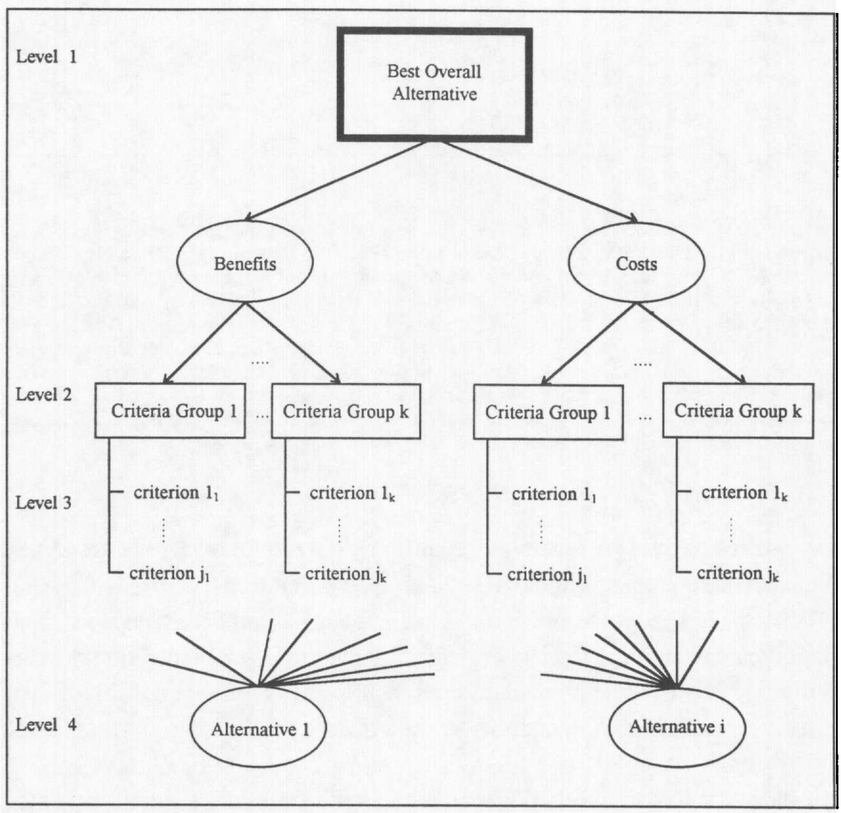

FIGURE 1. 4-Level hierarchy for AHP.

Non-Traditional Capital Investment Criteria, on the other hand, places the alternatives at level two, right below the main objective (Figure 2). For each alternative, the hierarchy is composed of categories (groups) containing the criteria which are at the final level of the hierarchy. NCIC does not require that the user differentiate between the benefits and costs in the hierarchy; instead, this can

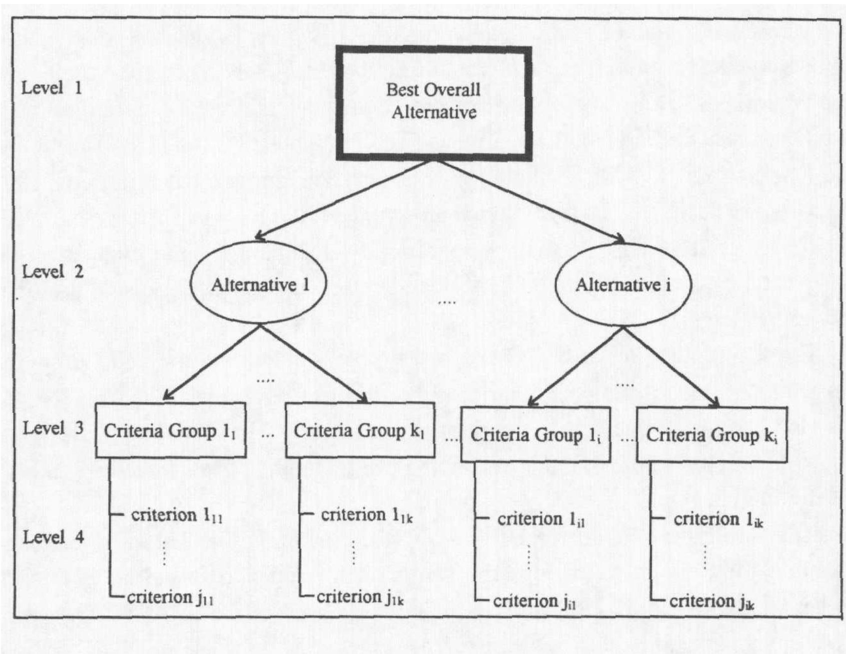

FIGURE 2. 4-Level hierarchy for NCIC.

be done in the questioning procedure. However, there is nothing to prevent a user from constructing both a benefits hierarchy and a cost hierarchy at level 3.

In both methods, data is obtained from the decision makers through pairwise comparisons among the elements at one level of the hierarchy with respect to an element in the next higher level. The magnitude of the response, which is a value between one and nine, or its reciprocal, is the strength of preference. In AHP, the pairwise comparisons are carried out separately for benefits and costs. Within each hierarchy there are three types of comparisons : (1) major categories are compared to each other, (2) criteria within these categories are compared to each other with respect to the categories, and (3) alternatives are compared to each other with respect to each criterion. The overall weight for each alternative is computed from the priority vectors of individual comparison matrices.

NCIC makes the pairwise comparisons in a somewhat different manner, interpreting the relative importance of criteria in monetary terms. In NCIC, comparisons are made only between the criteria within each major category and the criterion Annual Benefit (or Annual Cost) is incorporated into each pairwise comparisons matrix. Therefore, the resulting weights of criteria can be interpretlevel, the weights (or values) at the alternatives level are derived from the aggregate weights (or values) at the criteria level. Therefore, strictly speaking, NCIC is not a hierarchical model. The upper levels of the hierarchy are used simply to model the problem, not to execute the model.

There are examples of the application of AHP to the evaluation of advanced manufacturing technologies (Arbel and Seidmann [1], Chandra and Schall [12] and Wabalickis [22]). There are also case studies that address capital investment in other areas (Barbarosoglu and Pinhas [2]; Ghotb and Warren [15]; Tarimcilar and Khaksari [20]). AHP methodology has also been implemented in two different software packages: Expert Choice (Forman *et al.* [14]) and Automan (Weber [25]). Although Automan is specifically designed for the evaluation of manufacturing systems, Expert Choice can be used as a generic decision problem solving package.

For the application of NCIC to a real problem in automation and Computer Integrated Manufacturing (CIM), one may refer to Boucher *et al.* [6,8]. NCIC has also been implemented in a software package by MacStravic and Boucher [17, 26], which is commercially available through the Materials Handling Industry of America [18].

The similarities between AHP and NCIC have sometimes led to the conclusion that they are more or less the same. Thurston and Locascio [21] and Barbarosoglu and Pinhas [2] refer to NCIC as an application of AHP. One purpose of this paper is to clarify the differences that exist between the NCIC framework of analysis and the AHP methodology. It will be shown that these differences are quite significant vis-à-vis the capital investment decision problem.

In this paper we will also define, in a mathematical sense, the relationship between NCIC and AHP. In particular, it will be shown that a set of decision makers' numerical judgments in an AHP hierarchy have a one-to-one correspondence to a set of decision makers' judgments in an NCIC hierarchy. In effect, if you have the weights provided by the decision maker in one framework, it should be possible to predict how the decision maker will respond if the same decision problem is evaluated in the other framework. This raises some interesting empirical testing issues about which we shall elaborate later.

In the next section, the rationale behind the development of Non-Traditional Capital Investment Criteria will be explained. This includes a discussion of some of the shortcomings of AHP that NCIC was designed to address. Subsequently, we develop a set of equations that define the theoretical relationship between numerical outcomes in AHP and numerical outcomes in NCIC. Finally, we will give an application of the derived equations to a decision problem in automation and computer integrated manufacturing.

THE RATIONALE UNDERLYING NON-TRADITIONAL CAPITAL INVESTMENT CRITERIA

The main initiative in the development of NCIC was the necessity to tie the criteria weights to dollar values. While conducting the pairwise comparisons, one requirement of NCIC is that the known annual benefit (or annual cost) related to the alternatives be included in every matrix. This leads to the interpretation of the priority vectors for the pairwise comparisons matrices in terms of monetary amounts. This enables the decision makers to justify the final result of the analysis using the benefit and cost structure of the alternatives, which allows a net present value computation to be performed. It was argued in [9] that this brings the decision methodology within the capital investment framework, making it more familiar to corporate financial management.

Besides this main initiative, there were other difficulties of AHP that were also addressed by NCIC. One of these is the requirement of a single hierarchy of decision elements for the evaluation of all alternatives. This may not be appropriate in some cases. One criterion that has positive or negative value for an alternative may be completely irrelevant for another alternative. However, since there is one common hierarchy for all alternatives under AHP, there have been two possible solutions suggested to this problem. The first is to eliminate that criterion from the AHP hierarchy. If this is done, then the alternative exhibiting that criterion receives no credit for it. A second approach is to assign a judgment of maximum relative importance (9) to the alternative exhibiting that criterion. In this case, the alternative not exhibiting the criterion is given some credit for it (1/9). In any case, the credibility of the solution to the problem is affected.

In NCIC, however, this problem is solved by placing the alternatives at level 2 of the hierarchy, immediately after the global objective. Therefore, the hierarchy is divided into separate hierarchies for each alternative at level 2 and only the criteria that are relevant to each alternative are taken into consideration, solving the common hierarchy problem. A problem situation of this type is described in reference [8].

Another area in which the AHP has been criticized is the manner in which the criteria weights are elicited and assessed. The decision maker is asked questions such as: " With respect to category k, which is more important, criterion 1 or criterion 2, and by how much?" Belton [3] has argued that such questions are meaningless. She has pointed out the importance of having a clear understanding of the quantities of each criterion being compared.

Boucher and MacStravic [9] have shown that the suggestions by Watson and Freeling [23, 24] and Belton and Gear [4] that the criteria weights are a weighing of an average level of the criteria in AHP are correct. They have demonstrated that unless it is assumed that the decision makers must be thinking of some average quantities when making pairwise comparisons, the results obtained through AHP cannot be relied upon to be consistent with economic theory.

In NCIC, as a consequence of the hierarchy and the pairwise comparison procedure, the type of questions asked are of the form: "For alternative *i*, what is the relative importance of criterion 1 to criterion 2?" Therefore, the value of pairwise comparisons depend on the relative value of the criterion for a specific alternative.

Another well-documented problem with the AHP is that of rank reversal (Watson and Freeling [23], Belton and Gear [4], and Dyer [13]). It has been illustrated by numerical examples that the introduction of a new alternative into the decision problem can sometimes reverse the rankings of previously rated alternatives. In AHP, the criteria weights are assessed independently of the alternatives under consideration. Therefore, there is no relationship between the weights of alternatives with respect to criteria and overall criteria weights. When these unrelated weights are aggregated into a single measure to give a final ranking of alternatives, the problem of rank reversal may occur.

In NCIC, due to the structure of the decision hierarchy, each alternative is evaluated independently. The relative weights of criteria are determined by making pairwise comparisons of criteria at the levels exhibited by a given alternative. As a result, the introduction of a new alternative into the analysis does not change the criteria weights with respect to any one of the existing alternatives. Thus, in NCIC methodology, the problem of rank reversal is not encountered.

Another source of criticism is the way AHP handles benefits and costs. Bernhard and Canada [5] have discussed problems associated with the aggregation procedure of the benefits vector and the costs vector. To combine these two measures, Saaty [19] suggested computing the ratios of benefit and cost vector elements for each alternative and selecting the alternative with the highest ratio. However, Bernhard and Canada [5] have argued that unless an incremental analysis is conducted, this type of an analysis can lead the decision maker to a wrong conclusion.

In NCIC, as a result of interpreting all criteria weights in monetary terms, the alternatives have a priority vector in terms of total annual benefits and total annual costs. The net annual benefit can then be discounted to yield a present worth.

Another limitation we have noted in the AHP framework is the necessity of having at least two alternatives to evaluate in order to apply the methodology. There are situations in which there is only one investment to be considered and the issue is whether or not the future benefit of that investment is greater than its cost. When there are difficult to quantify criteria involved, a multiattribute methodology is still relevant. However, the AHP is difficult to use unless the user creates a fictitious alternative against which to compare. The NCIC methodology is able to handle the single investment alternative problem. An example of this kind of investment situation is given in reference [6].

Finally, it should be pointed out that financial management is not actually concerned with whether or not one alternative is superior to another in an investment decision. The central question of interest is whether or not the best alternative has benefits that exceed its cost. Those benefits may be weights or dollar denominated weights, but they should sum to a magnitude greater than the cost. Answering this concern is a feature of NCIC that is absent in the AHP.

Regardless of what procedure is used to gather and process judgmental information, the results or conclusions from applying the methodology is strictly a function of the judgments. Unlike the more objective approach of measuring the cost reductions or revenue benefits of a capital investment using traditional cost analysis, there is no measurement justification of a judgment. More specifically, the assumptions used in the traditional cost analysis are usually made explicit; the assumptions underlying a judgment usually are not. This leads to the issue of objective verification.

In the NCIC approach, we have promoted three ideas. The first is the use of "returns to scale" when comparing alternatives that exhibit different levels of a criterion and the criterion is measurable on some cardinal scale. An example of this is "lead time" which can be expressed in units of "days." Further discussion and examples of this appear in references [8, 9]. A second idea is to compare the resulting weight vector of criteria with the decision makers' holistic judgment concerning the ranking of criteria. It is well known that ordinal holistic judgments are relatively easy for individuals to make and are reasonable reflections of their beliefs. There should be a close correspondence between the ranking of criteria in the weight vector that results from applying the AHP or NCIC methodology and the ranking of criteria in the holistic judgment. Examples of this are shown in references [6, 8]. Another approach is to take the most significant (highest valued) criteria; i.e., those criteria that dominate the final conclusion, and to try to verify their value. This usually requires a cost model for those criteria. Examples of this are given in references [6, 8].

RANK REVERSAL AND THE "AVERAGE WEIGHT ASSUMPTION" REVISITED

Before developing the mathematical relationship between the AHP and NCIC, it is useful to revisit the arguements previously made in Belton and Gear [4] and Boucher and MacStravic [9]. These are the arguements that the criteria weights in the AHP must be a weighting of the average level of the value of the alternatives on each criteria and that any other assumption does not make sense from an economic perspective. We will borrow the example problem used in [4] to illustrate the case.

Consider the data shown in Table 1 concerning four alternatives (A,B,C,D) that are being evaluated on three criteria (a,b,c). The hypothetical values shown in the table are cardinal values. They represent value in terms of some common measure. In financial calculations these would be values in dollar terms. However, the measure could be utility or any other concept a decision maker might wish to use. The important point is that the data represents the absolute values of alternatives on criteria.

TABLE 1. Cardinal values for 4 alternatives on 3 criteria.

	Criteria			
Alternative	*a*	*b*	*c*	*Total*
A	100	900	800	1800
B	900	100	900	1900
C	100	100	100	300
D	900	100	900	1900

Neither the AHP nor NCIC explicitly require this data as input to the decision process. Both methodologies collect experts opinions as ratios of value (importance) as perceived by the decision makers. However, the decision makers have an implicit version of Table 1 in their minds from which these ratios emerge; otherwise, the ratios would not have meaning.

Let us assume that the Decision Maker wishes to compare alternatives A, B, and C only. In the case of the AHP, the pairwise comparisons matrices of alternatives with respect to criteria that would be given by a decision maker having the underlying cardinal values of Table 1 are shown in Figure 3. The three matrices are perfectly consistent and are computed by simply taking ratios of cardinal values from Table 1. In order to compute the overall weights of alternatives, it is first necessary to assign weights to the criterion. Following Belton and Gear [4], we assume the Decision Maker assigns equal weights; i.e., $(a,b,c)=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$. With these values as input, and using the AHP calculation of final weights, the final alternative weight vector of alternatives is (A,B,C) = (0.45,0.47,0.08). Therefore, B > A > C, and alternative B is preferred.

It is at this point that Belton and Gear introduce alternative D. This results in a new set of pairwise comparisons matrices as shown in Figure 4. Of course, the existing ratios among A, B, and C do not change; only the new relationships to D are added. Following the prior assumption that the criteria have equal weighting (importance), the new final weight vector is (A,B,C,D) = (0.37, 0.29,0.06,0.29). Therefore, A > B ~ D > C, and now alternative A is preferred.

w.r.t. Criterion a	A	B	C
A	1	1/9	1
B	9	1	9
C	1	1/9	1

w.r.t. Criterion b	A	B	C
A	1	9	9
B	1/9	1	1
C	1/9	1	1

w.r.t. Criterion c	A	B	C
A	1	8/9	8
B	9/8	1	9
C	1/8	1/9	1

FIGURE 3. Relative importance of alternatives A, B, and C on criteria a, b, c.

w.r.t. Criterion a	A	B	C	D
A	1	1/9	1	1/9
B	9	1	9	1
C	1	1/9	1	1/9
D	9	1	9	1

w.r.t. Criterion b	A	B	C	D
A	1	9	9	9
B	1/9	1	1	1
C	1/9	1	1	1
D	1/9	1	1	1

w.r.t. Criterion c	A	B	C	D
A	1	8/9	8	8/9
B	9/8	1	9	1
C	1/8	1/9	1	1/9
D	9/8	1	9	1

FIGURE 4. Relative importance of alternatives A, B, C and D on criteria a, b, c.

This is the "rank reversal" problem. Without any change in the relative importance among existing alternatives, the introduction of a new alternative has changed the preference order of the existing alternatives.

Why does this happen? The answer lies in the "average value assumption," which we will explain in a simple intuitive manner. The relative "importance," or "value" of each criteria must be related to the values of those criteria as exhibited by the alternatives. The relative values of the criteria as given by their weighting are not disassociated from the decision problem at hand. With regard to Table 1 and the choice among alternatives A, B, and C, the relative values of criterion a, b, and c can be computed from the values indicated in the columns. Thus, for criteria A, the total value represented by A + B + C = 1100. In Table 2 we have calculated the total values and relative weights of criteria for the cases of comparing three alternatives (A,B,C) and four alternatives (A,B,C,D). Note that they are not equally weighted and that the weights change as the alternatives under consideration change. If the AHP calculation is used to compute the final vector of alternative weights, allowing the appropriate criteria weights (as given in Table 2) to be applied in each case, the resulting final weight vector of alternatives is as follows:

$$(A, B, C) = (0.45,\ 0.475,\ 0.075)$$
$$(A, B, C, D) = (0.305,\ 0.322,\ 0.051,\ 0.322).$$

Note that the relation B > A is preserved. Further note that $\frac{A}{B} = \frac{0.45}{0.475} = \frac{0.305}{0.322} = 0.947$. In other words, the relationship between existing alternatives is preserved when a new alternative is introduced if the weights are properly adjusted. This is the insight provided by Belton and Gear [4] when they concluded that the weights used in the AHP must reflect an "average level" over the alternatives being considered. Furthermore, with reference to the last column of Table 1, it is clear that the relationship between overall value of A and B is given by the ratio $\frac{A}{B} = \frac{1800}{1900} = 0.947$, which is identical to the AHP calculation when weights are properly adjusted. If we assume that Table 1 represents values in dollars, it is clear that the only way the AHP can have economic meaning is if the average weight assumption is being made by the Decision Maker when giving criteria weights. This is the point made, more analytically, in Boucher and MacStravic [9, pages 13-15].

TABLE 2. Relative weight vector of criteria.

	a	*b*	*c*	*Total*
A+B+C	1100	1100	1800	4000
Relative criteria weights	11/40	11/40	18/40	
A+B+C+D	2000	1200	2700	5900
Relative criteria weights	20/59	12/59	27/59	

The argument just presented is the basis for the remainder of this paper. If the AHP is used for economic decision making in which criteria quantified in economic terms and difficult-to-quantify criteria are combined in arriving at the overall value of an alternative, one must assume that the Decision Maker is implicitly weighting the criteria based on the average value (importance) of the alternatives on that criteria. Otherwise, the final vector weights do not make economic sense. When it is assumed that the decision maker is doing this, a straightfoward mathematical relationship between the AHP and NCIC emerges.

THE MATHEMATICAL RELATIONSHIPS BETWEEN OUTCOMES IN AHP AND NCIC

Both the AHP and NCIC have been used to evaluate capital investments. It would be of interest to examine how results obtained by each methodology in actual decision problems compare to each other. To be able to compare the outcomes of the two methodologies, it is necessary to define a relationship between them. This relationship will be developed under the hypothesis that the AHP and NCIC are equivalent methods for eliciting a decision maker's preferences regarding capital investment decisions when the "average level assumption" is made with regard to criteria weights.

We will consider a three level hierarchy consisting of only benefits. For reasons of simplicity we will assume two alternatives being evaluated on two criteria. The results that will be derived for this simplified case will be generalized for problems with more than two alternatives and/or more than two criteria. A sample decision hierarchy with the notation for AHP is given in Figure 5.

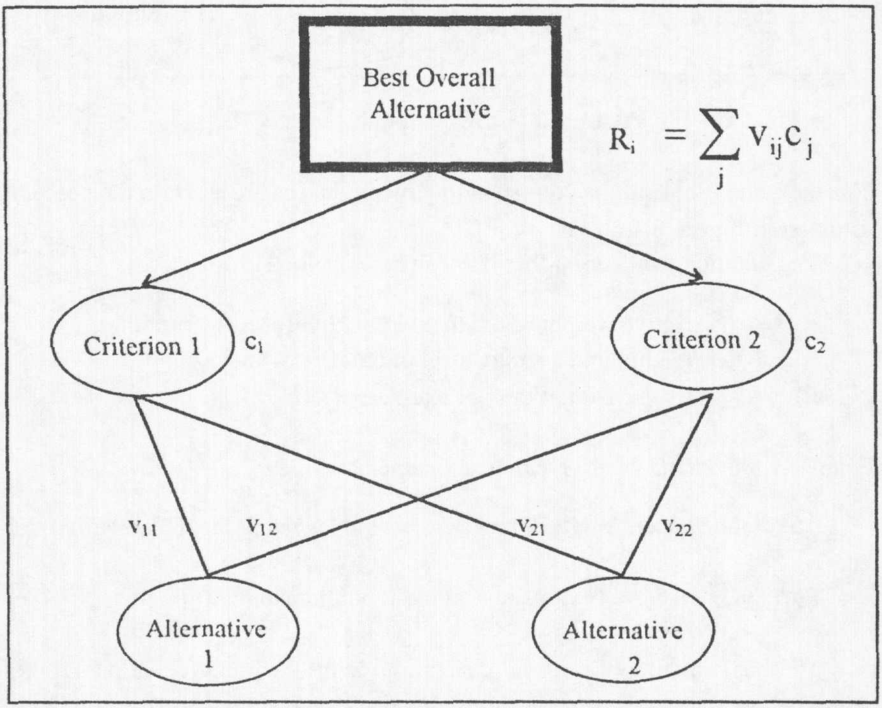

FIGURE 5. 3-Level hierarchy for AHP.

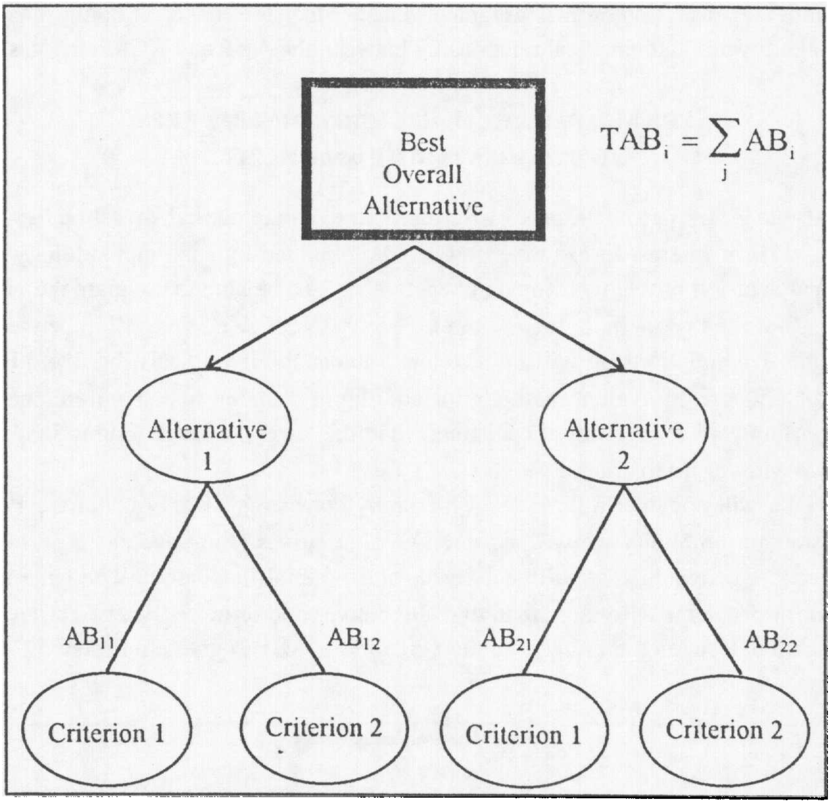

FIGURE 6. Level hierarchy for NCIC.

For the same problem, the corresponding hierarchy for NCIC, with the notation, is given in Figure 6.

The notation that is used for AHP can be explained as follows :

v_{ij} = the relative weight of alternative i with respect to criterion j,
c_j = the weight of an average level of criterion j, and
$R_i = \sum_j v_{ij} c_j$ = overall rank of alternative i.

Similarly, the NCIC notation can be explained as follows :

AB_{ij} = the annual benefit of criterion j at the level exhibited by alternative i,

and

$TAB_i = \sum_j AB_{ij}$ = Total Annual Benefit of alternative i.

The basic assumption in the derivation of the mapping equations comes from the fact that when using the AHP, the decision makers must be thinking of the sum or average of the value of alternatives with respect to criteria when making their judgments about criteria weights; that is, criteria weights are a weighing of the total or average level of the criteria over all alternatives in the Analytic Hierarchy Process (Watson and Freeling [23,24], Belton and Gear [4], and Boucher and MacStravic [9]). Thus the ratio of criteria weights should be equal to the ratio of the average value of the criteria. This assumption can be written as :

$$\frac{c_1}{c_2} = \frac{(AB_{11} + AB_{21})/2}{(AB_{12} + AB_{22})/2}$$

which simplifies to:

$$\frac{c_1}{c_2} = \frac{(AB_{11} + AB_{21})}{(AB_{12} + AB_{22})} \qquad (A1)$$

Since we are assuming that AHP and NCIC are equivalent methods of eliciting preferences, then the ratio of the contribution of criterion 1 to the overall rank of alternative 1 to the contribution of the same criterion to the overall rank of alternative 2 in both methods should be equal :

$$\frac{v_{11}c_1}{v_{21}c_1} = \frac{AB_{11}}{AB_{21}}$$

which simplifies to

$$\frac{v_{11}}{v_{21}} = \frac{AB_{11}}{AB_{21}} \qquad (A2)$$

Similarly, we can write :

$$\frac{v_{12}}{v_{22}} = \frac{AB_{12}}{AB_{22}} \qquad (A3)$$

In addition, because the priority vectors are normalized, the weights add up to 1. Therefore, the following equalities hold :

$$c_1 + c_2 = 1, \qquad (E1)$$

$$v_{11} + v_{21} = 1, \qquad (E2)$$

$$v_{12} + v_{22} = 1. \qquad (E3)$$

Using the above assumptions and equalities, we can develop the mapping equations.

NCIC → AHP CONVERSION EQUATIONS:

Solving the assumption (A1) and the equation (E1) together, we find the equations that map the Annual Benefits obtained by NCIC to criteria weights that would have been obtained by AHP. The variable $c_1{}'$ denotes the overall weight of criterion 1 as implied by NCIC outcomes.

$$c_{1'} = \frac{AB_{11} + AB_{21}}{AB_{11} + AB_{21} + AB_{12} + AB_{22}}, \text{ and} \tag{1}$$

$$c_{2'} = \frac{AB_{12} + AB_{22}}{AB_{11} + AB_{21} + AB_{12} + AB_{22}} \tag{2}$$

Similarly, we can solve assumption (A2) with equality (E2), and assumption (A3) with equality (E3) to obtain the relative weights of alternatives with respect to criteria. The variable $v_{11}{}'$ denotes the relative weight of alternative 1 with respect to criterion 1 as implied by the NCIC outcome.

$$v_{11'} = \frac{AB_{11}}{AB_{11} + AB_{21}} \text{ and } v_{21'} = \frac{AB_{21}}{AB_{11} + AB_{21}}, \tag{3}$$

$$v_{12'} = \frac{AB_{12}}{AB_{12} + AB_{22}} \text{ and } v_{22'} = \frac{AB_{22}}{AB_{12} + AB_{22}}. \tag{4}$$

Therefore, using the above equations we can compute the AHP weight vectors, both the weights of criteria and the weights of alternatives with respect to criteria, that are implied by the Annual Benefits obtained through NCIC.

It should also be noted that these conversion equations are consistent with AHP making the assumption that the category weights and criteria weights are an "average weight" as shown in the references [4, 9, 23, 24] and in the previous section. Equations (1) and (2) are based on the assumption that AHP criteria weights depend on an average dollar value of criteria given by an NCIC analysis.

AHP → NCIC CONVERSION EQUATIONS

We have seen how to map NCIC outcomes to a set of data comparable with the AHP outcomes. The next step is to generate the equations to calculate the implied annual benefits, a set of data comparable with the NCIC results, using the AHP weight vectors.

Definition: The contribution of criterion j to the overall rank of alternative i in AHP is :

$$\frac{R_i - R_i\left(v_{ij}c_j = 0\right)}{R_i}, \text{ and}$$

the contribution of criterion j to the Total Annual Benefit of alternative i in NCIC is :

$$\frac{TAB_i - TAB_i((AB_{ij} = 0)}{TAB_i}$$

Under the assumptions (A1) - (A3) we can show that the contribution of criterion j to the overall rank of alternative i in AHP is equal to the contribution of criterion j to the Total Annual Benefit of alternative i in NCIC. This assertion is proven next.

$$\frac{R_1 - R_1(v_{11}c_1 = 0)}{R_1} = \frac{v_{11}c_1}{v_{11}c_1 + v_{12}c_2} \qquad (5)$$

Substituting equations (1) - (4) which are direct consequences of the assumptions (A1) - (A3) into the above equation we have :

$$\frac{R_1 - R_1(v_{11}c_1 = 0)}{R_1} =$$

$$\frac{\dfrac{AB_{11}}{AB_{11}+AB_{21}} \cdot \dfrac{AB_{11}+AB_{21}}{AB_{11}+AB_{21}+AB_{12}+AB_{22}}}{\dfrac{AB_{11}}{AB_{11}+AB_{21}} \cdot \dfrac{AB_{11}+AB_{21}}{AB_{11}+AB_{21}+AB_{12}+AB_{22}} + \dfrac{AB_{12}}{AB_{12}+AB_{22}} \cdot \dfrac{AB_{12}+AB_{22}}{AB_{11}+AB_{21}+AB_{12}+AB_{22}}}$$

which simplifies to :

$$\frac{R_1 - R_1(v_{11}c_1 = 0)}{R_1} = \frac{AB_{11}}{AB_{11}+AB_{12}} = \frac{TAB_1 - TAB_1(AB_{11} = 0)}{TAB_1}$$

In general, this result can be written as :

$$\frac{R_i - R_i(v_{ij}c_i = 0)}{R_i} = \frac{TAB_i - TAB_i(AB_{ij} = 0)}{TAB_i} \qquad (6)$$

The equivalence of the contribution of a criterion to the overall rank of the alternative in AHP and the Total Annual Benefit in NCIC can be written as :

$$\frac{v_{ij}c_j}{\sum_j v_{ij}\; c_j (= R_i)} = \frac{AB_{ij}}{\sum_j AB_{ij} (= TAB_i)} \qquad (7)$$

Assume the "Annual Benefit" criterion to be criterion 1. Then, AB_{i1} will be the "Annual Benefit" of alternative i which is common to both AHP and NCIC. We can solve equation (7) to calculate the Total Annual Benefit of alternative i as implied by AHP by substituting $j = 1$:

$$TAB_{i'} = AB_{i1} \frac{\sum_j v_{ij} c_j}{v_{i1} c_1} \qquad (8)$$

Once the Total Annual Benefit of an alternative is known, by substituting it back into equation (7) the Annual Benefits of all other criteria as implied by AHP outcomes can be computed :

$$AB_{ij'} = TAB_{i'} \frac{v_{ij} c_j}{\sum_j v_{ij} c_j} = AB_{i1} \frac{\sum_j v_{ij} c_j}{v_{i1} c_1} \frac{v_{ij} c_j}{\sum_j v_{ij} c_j} \qquad (9)$$

$$AB_{ij'} = AB_{i1} \frac{v_{ij} c_j}{v_{i1} c_1} \qquad (10)$$

With equation (10) we have a mapping that allows us to generate the implied annual benefit of criteria with respect to each alternative using the vectors of criteria weights and alternative weights with respect to criteria obtained as a result of the AHP analysis.

Using equations (1) - (4) and (10) we can transform the outcome of one method into a set of data comparable with the outcome of the other method for a decision problem with a 3-level hierarchy. These results can easily be generalized for a 3-level hierarchy with multiple criteria and multiple alternatives as follows :

$$NCIC \rightarrow AHP: \quad c_{j'} = \frac{\sum_i AB_{ij}}{\sum_i \sum_j AB_{ij}} \quad and \quad v_{ij'} = \frac{AB_{ij}}{\sum_i AB_{ij}}$$

$$AHP \rightarrow NCIC: \quad AB_{ij'} = AB_{i1} \frac{v_{ij} c_i}{v_{i1} c_1}$$

where c_1 is the annual benefit criterion whose value is known to be AB_{i1}.

It is possible to apply statistical tests to the two sets of outputs once the data sets are comparable. In cases where both methods are applied using the same group of decision makers for the same problem, the transformation equations would provide a way to compare the outcomes of the two decision processes. In the next section we will apply the transformation equations to a multiattribute decision problem found in Canada and Suliivan [10].

EXAMPLE

We now demonstrate the use of these equations through a numerical example taken from Canada and Sullivan [10, page 262]. They use the AHP methodology for the comparison of three automation alternatives, P1, P2, and P3. The decision hierarchy consists of three levels and only benefit criteria. The hierarchy for the problem is given in Figure 7.

Using pairwise comparisons, the weights of the criteria and the weights of the alternatives with respect to each criterion are computed. The final weight vectors, as given by Canada and Sullivan [10], are illustrated in Table 3. The first row of data are the weights of the criteria and each column below that contains the weights of alternatives with respect to the corresponding criterion.

In order to be able to map these AHP weight vectors into a set of dollar amounts that represent the outcome of the corresponding NCIC problem we need to know the "Net Annual Benefit" of each alternative. Although the Net Annual Benefits are not given in dollar terms in Canada and Sullivan, this data is implicitly given in the weight vector for the alternatives with respect to criterion 1.

Assuming AB_{11}, AB_{21}, AB_{31}, are the "Net Annual Benefits" of alternatives P1, P2, P3, respectively, the weight vector states that:

$$\frac{AB_{11}}{AB_{21}} = \frac{0.12}{0.55},$$

$$\frac{AB_{11}}{AB_{31}} = \frac{0.12}{0.33}, \textit{ and}$$

$$\frac{AB_{21}}{AB_{31}} = \frac{0.55}{0.33}.$$

Therefore, for illustration purposes and without loss of generality, we can assume any total dollar amount for measured "Net Annual Benefits" and divide it among the alternatives. Assuming $10,000, we arrive at:

$$AB_{11} = \$1,200$$
$$AB_{21} = \$5,500, \textit{ and}$$
$$AB_{31} = \$3,300.$$

We apply these dollar amounts (AB_{i1}'s) and c_j's and v_{ij}'s that are given in Table 3 to equation (10) to transfer the AHP weight values (c_j's and v_{ij}'s) into a set of dollar values that would be obtained as a result of the corresponding NCIC problem $\left(AB'_{ij}\text{'}s\right)$. These resulting dollar amounts associated with each criterion

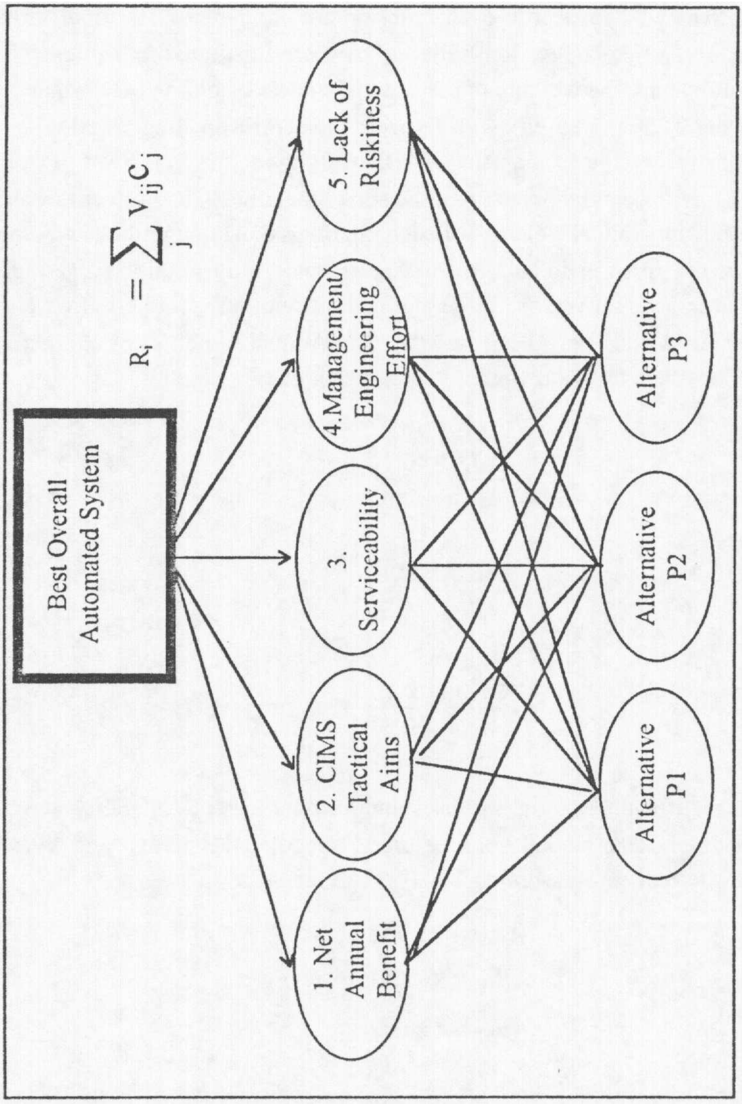

FIGURE 7. 3-Level AHP hierarchy for the numerical example.

TABLE 3. Weights of criteria and alternatives with respect to each criterion.

	Criteria				
	1. Net Annual Benefit	2. CIMS Tactical Aims	3. Serviceability	4. Management/ Engineering Effort	5. Lack of Riskiness
Criteria Weights (c_j)	0.489	0.288	0.086	0.041	0.096
Alternative Weights (v_{ij})					
P1	0.12	0.21	0.50	0.63	0.62
P2	0.55	0.55	0.25	0.30	0.24
P3	0.33	0.24	0.25	0.07	0.14

for each alternative are given in Table 4. The entries of Table 4 are the values that the same decision maker who provided the weights of Table 3 would be expected to provide if her/his pairwise comparisons were taken within the NCIC framework.

For completeness of exposition, we will map these NCIC dollar amounts that are implied by the AHP weight vectors back to AHP weight vectors. Hence, assuming that the data in Table 4 is the result of the NCIC analysis of the problem, we will find the AHP weight vectors as implied by these NCIC dollar amounts. If the equations we derived for mapping NCIC results to AHP results and AHP results to NCIC results are all correct, we should end up with the original weight vectors given in Table 3. To map the NCIC dollar amounts (AB_{ij}'s which are the AB_{ij}''s of Table 4) into AHP weight vectors (v'_{ij}'s and c'_j's) we make use of equations (1) - (4) and obtain the data given in Table 5.

As can be seen from the data of Table 5, the AHP weight vectors as implied by the NCIC dollar amounts are exactly equal to the original AHP weight vectors. Hence, when we start with the resulting data set of one method, map it into the outcome of the other method using the equations we derived and then map these implied values back into the outcome of the first method, we end up with the original data set that we started with. Therefore, besides showing an application of the mapping equations, this numerical example demonstrates that these equations (Equations (1) - (4) and (10)) provide correct transformations between the AHP and the NCIC outcomes under the "average level assumption."

In cases where both methods are applied using the same group of decision makers for the same problem, the transformation equations would provide a way to compare the outcomes of the two decision processes. It is possible to apply statistical tests to the two sets of outputs once the data sets are comparable.

CONCLUSIONS

In this study we investigated the relationships between two multicriteria decision making tools: The Analytic Hierarchy Process and Non-Traditional Capital Investment Criteria. We have developed and illustrated the use of the transformation equations that are necessary to map the outcome of one method into a data set that is directly comparable with the outcome of the other.

We began this paper by discussing some of the difficulties we perceive in the AHP method when it is applied to capital budgeting. These difficulties were the motivation for developing the NCIC methodology. However, putting theoretical issues aside, the most important practical reason for selecting an evaluation procedure is that it will provide the decision maker with a tool that yields conclusions that are consistent with what the decision maker believes. It is also important to test those conclusions against some objective reality.

TABLE 4. NCIC dollar amounts as implied by the AHP weight vectors.

Criterion	Alternatives			Total ($\sum_i AB'_{ij}$)
	P1	P2	P3	
1	$ 1,200	$ 5,500	$ 3,300	$ 10,000
2	$ 1,237	$ 3,239	$ 1,413	$5,889
3	$ 879	$ 440	$ 440	$ 1,759
4	$ 528	$ 251	$ 59	$ 838
5	$ 1,217	$ 471	$ 275	$ 1,963
Total ($TAB'_i = \sum_j AB'_{ij}$)	$ 5,061	$ 9,901	$ 5,487	$ 20,449

TABLE 5. AHP weight vectors as implied by the NCIC dollar amounts of Table 4.

	Criteria				
	1. Net Annual Benefit	2. CIMS Tactical Aims	3. Serviceability	4. Management/ Engineering Effort	5. Lack of Riskiness
Criteria Weights (c'_j)	0.489	0.288	0.086	0.041	0.096
Alternative Weights (v'_{ij})					
P1	0.12	0.21	0.50	0.63	0.62
P2	0.55	0.55	0.25	0.30	0.24
P3	0.33	0.24	0.25	0.07	0.14

One of the fundamental and important contributions of the AHP is the ability to measure the internal consistency of a decision maker's judgements (pairwise comparisons) using the maximum eigenvector approach. The equation sets developed in this paper offer yet another way to empirically investigate a decision maker's ability to use intuition (expert judgement) in dealing with complex decision problems. Put bluntly, if there is any validity to the use of human judgment, or intuition, in the solution of capital budgeting problems, that human judgment should show a certain amount of consistency in opinion, however the judgmental data is collected. We hope that the ability to map outcomes between two comparable decision methodologies will encourage empirical research in the direction of clarifying this issue.

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